Exercise 8

For each of the following integral equations, classify as Fredholm, Volterra, or Volterra-Fredholm integral equation and find its kind. Classify the equation as singular or not.

$$\frac{1}{2}x^2 - \frac{2}{3}x + \frac{1}{4} = \int_0^1 (x-t)u(t)\,dt$$

Solution

This is a Fredholm integral equation because both of the limits of integration are constant. It is of the first kind because the unknown function u appears only inside the integral. It's inhomogeneous because of the $(1/2)x^2 - (2/3)x + 1/4$ on the left side. It's not singular since neither of the limits of integration are infinite and the integrand does not become infinite in the interval of integration.